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An investigation into the best method to combine national and local data to develop site-specific critical loads

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Steve Killeen

Head of Science

EXECUTIVE SUMMARY

Critical loads provide national or European-wide maps of the sensitivity of habitats to acidification or eutrophication which are used in effects-based policy development. Although such maps are appropriate for national-scale assessments, this may not be the case for site-specific assessments due to assumptions and the resolution of the underlying datasets. At the site level, insufficient data or effects criteria generally exist to calculate critical loads for specific habitats or species.

The aim of this study was to present a new and alternative method for carrying out site-specific acidity critical loads assessments based on developing the relationships between vegetation communities, soil types and critical loads. This has been achieved by the application of Endorsement Theory – a non-numeric, qualitative approach. It provides an endorsement based on the 'weight of evidence' for different critical load values for the vegetation communities of the National Vegetation Classification (NVC), which are often used to describe the habitats of designated sites. In addition, a Dempster–Shafer formalism is proposed that combines and refines the estimates of critical loads derived from both the national maps and from the endorsement.

The full methodology is described in this report, which is accompanied by a Microsoft® Access database containing the underlying datasets, endorsement tables and queries to automatically generate summary or critical load endorsement reports for any selected NVC community. Aquatic vegetation communities are not covered by this study.

The report discusses the results in the context of the 22 heathland NVC communities in the UK. Heathland communities of one sort or another exist from the Lizard in Cornwall to the Brecklands of East Anglia to the northern isles of Scotland. This initial assessment suggests that the Endorsement Theory approach will provide the Environment Agency with a defensible, useable and useful tool. Uncertainties in the approach and areas requiring further research are identified.

CONTENTS

1. INTRODUCTION

There is an increasing demand from conservation agencies and policy makers for site-specific critical loads, particularly in response to the need to assess designated sites under the Habitats Directive in terms of their ‘favourable status’.

Calculating a critical load of acidity requires detailed knowledge of parameters, including the mineralogy and weathering rate of the soil, that are difficult to measure. National estimates of critical loads arose from the need to be able to map and quantify the potential for adverse effects from acidification and eutrophication to sensitive habitats. Such maps have been used both nationally and internationally to provide a strategic overview of the areas ‘at risk’ and the ability to investigate alternative policy options at the highest level (EU Directives and UNECE Protocols).

The methods and data used to calculate critical loads have been continually reviewed and refined since the late 1980s (see http://critloads.ceh.ac.uk). They remain a tool primarily designed for national assessment, however, and the use of national maps to assess a particular site can lead to counter-intuitive or misleading results.

There are few sites where the key parameters are measured directly. Thus, it is not generally possible to simply replace the national estimate with a local estimate. In addition, the amount of information available locally is variable; the soil and vegetation types may be recorded but the location of the designated feature(s) and its association with particular soil types unknown.

Fortunately, the vegetation communities of the National Vegetation Classification (NVC) (Rodwell, 1991) appear to be commonly used to describe the vegetation present within designated sites. Hence, we have targeted our methods at NVC communities and assume that the designated feature(s) can be associated with one or more of these.

The approach adopted by this study to define site-specific critical loads is to:

- develop the relationships between soil types and vegetation communities, and apply Endorsement Theory techniques to determine the most likely critical load class (work package 1);
- revise the national estimate of the critical load using as much local information as is available and by applying Dempster–Shafer methods (work package 2).

Endorsement Theory (Cohen, 1985; Sullivan and Cohen, 1985) is about reasoning with incomplete and uncertain evidence. Dempster–Shafer (Dempster, 1967; Shafer, 1976) is an extension of Bayesian statistics and is concerned with whether a hypothesis is *provable* following the addition of new uncertain evidence.

2. WORK PACKAGE 1: APPLICATION OF ENDORSEMENT THEORY TO CRITICAL LOADS

There are relatively few sites in the UK where all the appropriate information needed to calculate a critical load has been measured. To estimate a site-specific critical load, it is therefore necessary to convert or translate surrogate data. In this case the primary surrogate data are found in the National Vegetation Classification and the environmental data it contains are very variable between communities.

Endorsement Theory (Cohen, 1985; Sullivan and Cohen, 1985) is a form of reasoning with symbolic logic, i.e. it is a non-numerical approach. Most of the developments in Endorsement Theory have been made in the field of Artificial Intelligence, especially in attempts to develop expert systems that work in an analogous fashion to human experts. Endorsement Theory is especially useful for combining incomplete and uncertain evidence, in particular:

- it allows inference to be made from partial knowledge;
- the reasoning process is explicit, traceable and highly heuristic;
- it can represent common knowledge in a natural form and it is not necessary to translate an expert's' opinion into a numerical value.

We have used an Endorsement Theory approach to determine the appropriate soil acidity critical load class for NVC communities, based on:

- information available on acidity critical loads for different soil types in England and Wales;
- soil and ancillary information available for NVC communities.

Often the information is variable and incomplete. For each NVC community, we considered each of the six critical load classes (see Section 2.2.1) as an independent hypothesis and, using Endorsement Theory, we determined the extent to which the available data support each of those hypotheses.

The method is based on the following assumptions:

- The 'feature' for which a site is designated or identified for investigation can be associated with the vegetation cover, either through a direct observed relationship or by inference.

- The vegetation cover of the site has been, or can be, characterised in terms of National Vegetation Classification communities (Rodwell, 1991).

- The description of the environmental conditions associated with each NVC community can be related to the description of soils given by Loveland (1991) and to the description of soils given in the Soil Survey of England and Wales (Rothamsted, 1983).

- The relative frequency with which soil characteristics are associated with a critical load class is a measure of the 'weight of evidence' it provides for a particular hypothesis.

The following stages were used in applying the Endorsement Theory approach:

- Tabulate the environmental information about each NVC community in a Microsoft® Access database.

- Tabulate the information given by Loveland (1991) about each soil association in the Access database.

- Tabulate the information about the soil group, sub-group and type for each soil association described by Loveland (1991) in the Access database.

- Generate a series of look-up-tables (LUT) to translate between the NVC terminology and the Loveland/Soil Survey terminology.

- Devise rules that relate the relative and absolute frequency with which data items (terms) are associated with critical load classes to a 'weight of evidence'.

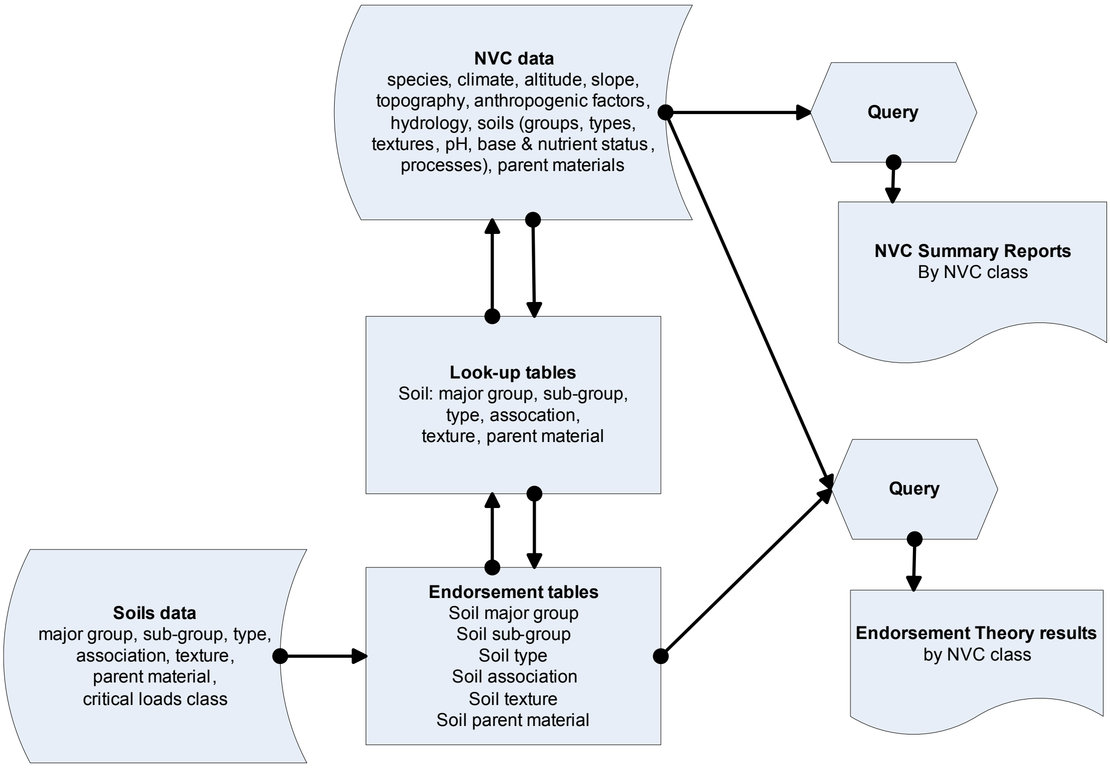
- Devise rules for combining all the evidence to provide varying degrees of 'endorsement' for each hypothesis.

- Devise database queries to relate and extract the endorsement evidence for each NVC community.

- Devise database queries to provide summary reports describing the environmental data for each NVC community.

Figure 2.1 summarises the flow of data and processes. The different stages are described below.

Figure 2.1 Summary of data structure and flow within the Microsoft Access database



2.1 The National Vegetation Classification system

The National Vegetation Classification has become the de-facto standard description of the vegetation communities or assemblages of the UK. Many plant species are strongly influenced by soil type and plants form a major component of the environment. It is assumed that the vegetation communities present on the site of interest are either already known or can be found relatively easily.

The descriptions of the NVC communities include a variable amount of information about the environment within which they occur. The description of each terrestrial NVC community (ie, NVC class) was read and the relevant information extracted and tabulated within the Access database. Purely aquatic communities and salt marsh communities were not processed.

The NVC communities examined and the types of data extracted are listed in Tables 2.1 and 2.2, respectively. Separate tables were set up in the Access database for each of the data types listed in Table 2.2, with relationships generated between all tables by NVC community.

There may be multiple entries in the database for a number of the parameters listed in Table 2.2 for some communities. While there may be information for all or most of the data types for some communities, there are relatively few for others. Examples are given in Annexes 1 and 2, which contain NVC Summary Reports (generated from the Access database) for two of the heathland communities (H1 and H21).

Table 2.1 NVC communities considered in this study

NVC communities*	Code letter	Number
Heathlands	H	22
Mires	M	38
Swamps	S	28
Upland and montane habitats	U	21
Mesotrophic grasslands	MG	13
Calcareous grasslands	CG	14
Woodlands	W	25
Total		**161**

*Aquatic communities were excluded on the basis that a different critical load methodology is applied to UK surface waters compared with terrestrial habitats.

Table 2.2 Environmental data extracted from NVC class descriptions

Data type	Parameters
Distribution	• General description
Climate	• General description • Annual rainfall • Annual number of wet days • Rainfall pattern • Maximum summer temperature • Minimum February temperature • Number of frosts per year
Altitude	• Minimum • Maximum • Mean • Additional comments
Slope	• Minimum • Maximum • Mean • Additional comments
Topography	• Itemised comments
Anthropogenic factors	• Itemised comments
Soils	• Soil association • Soil type • Additional comments
Soil texture	• Itemised comments
Base status	• Itemised comments
Nutrient status	• Itemised comments
Soil pH	• Minimum • Maximum • Mean • Additional comments
Soil processes	• Itemised comments
Parent material	• Itemised comments
Geological processes[1]	• Itemised comments
Hydrology	• Itemised comments
Species[2]	• Full species list • Frequency

[1] Data available for very few NVC communities and subsequently not used.

[2] Digital species list provided courtesy of David Roy, Ecological Processes Modelling Section, CEH Monks Wood.

2.2 Soils data

Unfortunately, suitably detailed information about soils (associations or series) is not readily available for many designated sites. Soil surveys are expensive and it is difficult for a non-expert to identify a soil series reliably from field observations.

Soil surveyors working in the field typically use changes in vegetation type in combination with topographic factors to estimate the location of boundaries between soil types. These boundaries are typically very diffuse but it is convenient to portray the boundaries on a map as if they were 'crisp'.

Two sources of soils information were used in this study:

- the report by Loveland (1991) describing a methodology for assigning acidity critical load classes to each soil association in England and Wales on the basis of their mineralogy and weathering rate;

- the Soil Survey of England and Wales 1983 legend for the 1:250,000 soil map (Rothamsted, 1983). Soil survey information in this series is now held by the National Soils Resources Institute (NSRI) at Cranfield University's Silsoe site (http://www.silsoe.cranfield.ac.uk/nsri/).

Because both sources cover only England and Wales, the methodology proposed relates only to these regions of the UK. The way in which the soils information in these two documents was used in the study is described below.

2.2.1 Soils information from Loveland's report

The report by Loveland (1991) documents the soil acidity critical loads methodology agreed by the Soils Working Party of the Department of Environment's (as it was then) Critical Loads Advisory Group. The report allocates 298 distinct soil associations (including 'man made soils') in England and Wales to one of six critical load classes (Table 2.3).

The methodology involves :

- taking the critical load classes as defined at the United Nations Economic Commission for Europe (UNECE) workshop in Skokloster in 1988 (Nilsson and Grennfelt, 1988) as a starting point;
- modifying those values up or down in the light of more specific information on the mineralogy, weathering, depth, geology, texture, etc. appropriate to the soil association.

A soil association will often contain several soil series and the different soil series in an association can have very different physical properties. The critical load was therefore set according to the properties of the dominant series within each association. The methodology (but not the data) as subsequently applied to the UK is described in more detail by Hornung *et al.* (1995).

Table 2.3 shows the classes of acidity critical loads for soils used in this study.

Table 2.3 Classes of acidity critical loads for soils

Critical load class[1]	**Critical loads range (keq ha^{-1} year^{-1})**	**Sensitivity to acidification**
CL-1	≤0.2	High sensitivity
CL-2	> 0.2 ≤0.5	
CL-3	> 0.5 ≤1.0	
CL-4	> 1.0 ≤2.0	
CL-5	> 2.0	Low sensitivity
CL-6 or peat	Peat soils[2]	

[1] These are the critical load classes as labelled in later sections of this report and as used in the Access database. In Loveland (1991), the classes are reversed with class 5 representing the most sensitive case.

[2] The assignment of critical loads on the basis of mineralogy and weathering rate is inappropriate for peat soils. Hence, all peat soils were assigned to a separate class and a different methodology used to determine their critical loads (Skiba and Cresser, 1989; Calver, 2003; Calver *et al.*, 2004).

In addition to critical load class, Loveland also contains information on the geology, mineralogy, texture and typical land use for each soil association together with a short 'key word' comment. Sometimes multiple terms are provided; where they are separated by a slash (/), the first term is dominant. This is illustrated in the example shown in Figure 2.2.

The information from the Loveland report relating the name of the association with the final critical load class, geology, texture and comment were extracted and stored in the Access database.

Information on the mineralogy is important. However, it is even less widely available than information about the named soil series or association and is not considered further.

Information on the land use is of interest in general to provide trends across a region. However, land use of a specific site – especially one protected for conservation reasons –is influenced by many other factors and is therefore not considered further

Figure 2.2 Example of information from Loveland report

As an example for the Soil Association Worcester (431)

Geology:	Permo-Trias red mudstone
Mineralogy:	chlorite / carbonates
Texture:	clayey / fine loam
Land use:	stock rearing / arable
Comment:	slow drainage

2.2.2 Information from the 1:250,000 soils map legend

The critical load classes assigned by Loveland (1991) relate to the soil associations of the Soil Survey of England and Wales 1:250,000 map legend (Rothamsted, 1985). The national map of acidity critical loads for soils is based on the dominant soil association in each 1-km grid square as defined by NSRI 1:250,000 scale data.

The soils of England and Wales are described by a hierarchical classification. This consists of 10 major soil groups, each of which is further divided into a number of soil sub-groups. Where these sub-groups are divided, we have referred to them as ‘soil types’. The individual soil associations fall within the ‘soil types’. For example, the Worcester soil association (431) featured in Figure 2.2 belongs to the following soil groups:

- Major group 4: pelosols;
- Sub-group 4.3: argillic pelosols;
- Soil type 4.3.1: typical argillic pelosol.

Using this classification, the soil group information was assigned to each soil association for England and Wales and stored within the Access database. This enables it to be directly related to the other soils information derived from Loveland (1991).

2.3 Relating NVC communities to soils terminology

The next stage in the process was to relate the environmental information extracted for the NVC communities to the soils (and critical loads) information from Loveland.

In some cases, there is an exact correspondence between a term used in the NVC description and a term used by Loveland or the Soil Survey of England and Wales. In many more cases, the terms are similar but not exactly the same, e.g. ‘brown soil’ and ‘brown earth’. In other cases, the terms are less similar or there appears to be no corresponding terms. For example, the description of nine very varied NVC communities refers to the ‘Borrowdale volcanics’ geology in the Lake District but this is not mentioned at all by Loveland in his geology descriptions.

A series of look-up-tables (LUT) were derived. These link the information for the NVC communities to the soil and geology terms derived from Loveland and the Soil Survey, which is necessary for the application of the endorsement methodology. These LUTs are included in the Access database. A parsimonious approach has been adopted; when in doubt, we have not inferred a connection. This issue is discussed further in Sections 4 and 5.

2.4 Devising the rules for 'weight of evidence'

Seven 'endorsement' tables (ET) relate the environmental factors to a critical load class. Four of the ETs are derived directly from information in Loveland:

(i) soil association
(ii) geology
(iii) texture
(iv) comment

The final three are based on the Soil Survey classification of each soil association as described in Section 2.2.2:

(v) major soil group
(vi) soil sub-group
(vii) soil type

The ET for soil association simply gives the critical load class as assigned by Loveland. The other ETs are based on the number of soil associations in England and Wales by a particular characteristic, ie, soil sub-group, soil texture, etc. For example, Table 2.4 shows the ET for the 32 soil sub-groups.

Table 2.4 Endorsement table for soil sub-groups

Soil sub-group	Number of soil associations by soil sub-group and critical load class[1]						Total
	CL-1	CL-2	CL-3	CL-4	CL-5	Peat	
alluvial gley		1		9	10		20
argillic brown earth	2	3	28	2	14		49
argillic gley		2	3				5
argillic pelosols					1		1
brown alluvial			4				4
brown calcareous alluvial					2		2
brown calcareous earth					17		17
brown calcareous sand					1		1
brown earth		9	19	1	4		33
brown podzol	1	6					7
brown sand		12					12
calcareous pelosols					4		4
cambic gley			3		1		4
disturbed						2	2
earthy peat						7	7
gley podzol	7						7
humic gley	3				3		6
humic-alluvial gley	1				2		3
humic-sandy gley	2						2
non-calcareous pelosols		1	1				2
paleo-argillic brown earth	1	3	6		2		12
podzol	8						8
ranker	1	4			3		8
raw peat						4	4
rendzina-like alluvial					1		1
rendzinas					15		15
sand-parendzinas					1		1
sandy gley		1	1				2
stagnogley		1	9	30	3		43
stagnohumic gley		5					5
stagnopodzol	4	3					7
unripened gley			1				1

[1] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads

Table 2.4 shows that there are 20 soil associations which are considered to be 'alluvial gleys'. Most of them have a high critical load and half (10) are in the highest class, but there is one alluvial gley which has a relatively low critical load (CL-2: range 0.2–0.5 keq ha^{-1} $year^{-1}$). All 12 of the soil associations described as 'brown sands' and all 15 of the 'rendzinas' are in a single class, but the 49 soil associations termed ‘argillic brown earth’ occupy all critical load classes except the peat class.

The question is to what extent does knowledge about the soil sub-group provide evidence for any particular critical load class. The solution is a two stage process:

- decide how many classes of ‘weight of evidence’ to use;
- determine the relationship between the numbers and the ‘weights’, e.g. in this case, between the number of soil associations per critical load class and the weight of evidence.

We decided to use four categories of weight of evidence:

- strong
- moderate
- weak
- very weak.

It is important to realise that these categories are based on the perception of an individual expert. Other experts may prefer more or fewer categories, or apply different labels and different divisions of the attribute space. In our case, we started with three classes but later decided to divide the ‘weak’ class into ‘weak’ and ‘very weak’. In the context of land cover change, Comber *et al.* (2003) also use four levels; however, they apparently provide greater detail at the ‘strong’ end of the spectrum as they use ‘conclusive-belief’, ‘prima-facie’, ‘strong’ and ‘weak’.

Having decided to use four classes of weight, we then needed to determine a set of rules which convert the row totals and cell values into our weight categories. For example, four out of seven stagnopodzols (57.1 per cent) and 19 out of 33 brown earths (57.6 per cent) are in one critical load class. But are these equivalent situations? Because there are only seven stagnopodzols and six potential critical load classes, we decided that the observation that four out of the seven are in one class provides ‘weak’ evidence (see below). On the other hand, for the brown earths we decided that 19 out of 33 provides a ‘moderate’ belief that the attribution is appropriate.

Every expert will have a different view on how many cases are needed before a particular observation provides a sufficient amount of evidence (for example, see Annex 5). With six options of critical load class in this case, we decided that when there are more than 10 cases of an item a simple arithmetic rule would be used (see Table 2.5).

Table 2.5 Weights assigned to attributes with a row total of more than 10

Cell value (where row total >10)	Weight
>0	very weak
≥25 per cent of total	weak
≥50 per cent of total	moderate
≥75 per cent of total	strong

When there are fewer examples, the situation is more complex. If there is only one case of something occurring (e.g. in Table 2.4, argillic pelosols, unripened gley, sand-parendzinas etc.), it has to fall into one cell and therefore does not add much weight to any attribution.

We decided that, where there are less than 10 cases of an attribute, the weights shown in Table 2.6 would be used. So, in the case of the stagnopodzol where there are seven cases in total, the four in the lowest critical load class provide 'weak' evidence for that hypothesis; there is also 'weak' support (three out of seven) for the hypothesis of the second lowest critical load class.

We can now combine the expert rules provided in Tables 2.5 and 2.6, and apply them to the data in Table 2.4 to provide a weight of evidence for any particular critical load class (Table 2.7).

Table 2.7 shows that, according to the rules we have adopted. only six soil sub-groups provide 'strong' evidence and a further seven soil sub-groups provide only a 'very weak' attribution. Different experts may provide a different view of the data, but any expert who is internally consistent will end up with a correlated result.

Table 2.6 'Weight of evidence' for rare features (≤10 cases in total)

Cell value	Row total (total number of cases) 1	2	3	4	5	6	7	8	9	10
1	very weak	very weak	very weak	very weak	very weak	very weak	very weak	very weak	very weak	very weak
2		weak	weak	very weak	very weak	very weak	very weak	very weak	very weak	very weak
3			moderate	weak	weak	weak	weak	weak	very weak	very weak
4				moderate	moderate	weak	weak	weak	weak	weak
5					moderate	moderate	moderate	moderate	weak	weak
6						strong	moderate	moderate	moderate	moderate
7							strong	strong	strong	moderate
8								strong	strong	strong
9									strong	strong
10										strong

Table 2.7 Endorsement table for soil sub-groups showing weight of evidence class[1]

Soil sub-group	Critical load class[2]					
	CL-1	CL-2	CL-3	CL-4	CL-5	Peat
alluvial gley		vw		w	m	
argillic brown earth	vw	vw	m	vw	w	
argillic gley		vw	w			
argillic pelosols					vw	
brown alluvial			m			
brown calcareous alluvial					w	
brown calcareous earth					s	
brown calcareous sand					vw	
brown earth		w	m	vw	vw	
brown podzol	vw	m				
brown sand		s				
calcareous pelosols					m	
cambic gley			w		vw	
disturbed						w
earthy peat						s
gley podzol	s					
humic gley	w				w	
humic-alluvial gley	vw				w	
humic-sandy gley	w					
non-calcareous pelosols		vw	vw			
paleo-argillic brown earth	vw	w	m		vw	
podzol	s					
ranker	vw	w			w	
raw peat						m
rendzina-like alluvial					vw	
rendzinas					s	
sand-parendzinas					vw	
sandy gley		vw	vw			
stagnogley		vw	w	m	vw	
stagnohumic gley		m				
stagnopodzol	w	w				
unripened gley			vw			

[1] vw = very weak; w = weak; m = moderate; s = strong.

[2] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads.

2.5 Combining the evidence to produce an endorsement

Having allocated a symbolic value (weight of evidence) to each piece of evidence describing the critical loads for an NVC community, these need to be combined to produce an overall endorsement.

The number of levels of endorsement and the exact terms used are again dependent on the view of the individual expert. In this case, we chose five levels of endorsement. These are, in descending rank:

- ‘Definitive’ – three or more pieces of ‘strong’ evidence and no conflicting evidence for other critical load classes.
- ‘Confident’ – two or more piece of ‘strong’ evidence and no ‘strong’ evidence for any alternative critical load class.
- ‘Likely’ – at least one piece of ‘strong’ evidence (other alternatives may have ‘strong’ evidence).
- ‘Weak’ – at least one ‘moderate’ or two ‘weak’ pieces of evidence (other alternatives may have greater weight).
- ‘Very weak’ – some evidence.

Once the strength of the evidence has been determined based on the information for an NVC community, the appropriate range of soil acidity critical loads can be defined.

Annex 1 describes the heathland community H1. This shows that, for some attributes, there are multiple entries, e.g. there are two references to major soil group (both brown soils) and three to the geology type. Each ‘statement’ has been given equal weight, related to the appropriate endorsement table(s) and, where possible, included in the estimation of the overall endorsement. This can give the impression of double counting, but this is not the case. Table 2.8 summarises all the evidence for H1 that could be related from the NVC to Loveland.

Table 2.8 Endorsement summary for H1 *Calluna vulgaris – Festuca ovina* heath[1]

ET consulted	'unified' terminology	Critical load class[2]					
		CL-1	CL-2	CL-3	CL-4	CL-5	Peat
Major soil group	Brown soil Brown soil	vw vw	w w	w w	vw vw	w w	
Soil sub-group	Brown earth Brown sand		w s	m	vw	vw	
Soil association	Newport Worlington		s s				
Texture	Sand Sand	m m	w w			vw vw	
Parent material	Drift (with sandstone or peat) Sand/sandstone Sand/sandstone	w m m	w w w	vw	 vw vw	vw	
Totals	very weak	2	0	1	5	4	
	weak	1	8	2	0	2	
	moderate	4	0	1	0	0	
	strong	0	3	0	0	0	
Final endorsement		weak	confident	weak	very weak	weak	

[1] vw = very weak; w = weak; m = moderate; s = strong.

[2] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads.

The 'unified' terminology is the term used in estimating the correspondence between the terms used by Rodwell (1991) and Loveland (1991), and given in the LUT (Section 2.3). For example, in this case the information on texture in Annex 1, 'sandy' and 'sandy-skeletal' are both deemed as being equivalent to 'sand' in the texture ET.

In this case, we can 'confidently' endorse the attribution that the H1 heath should be treated as having a critical load of acidity in the range 0.2 to 0.5 keq ha^{-1} year^{-1} (i.e. CL-2). There is no evidence to support a peat critical load for this community, but all other hypotheses are weak or very weak.

Applying the same methodology to H21 heathland reveals some problems. Not only is there much less evidence, but one piece of evidence is counter-intuitive (see Table 2.9). Rankers are classified as lithomorphic soils. According to the 1:250,000 soil map legend (Rothamsted, 1983), this major soil group consists of eight soil associations classified as rankers and a further 17 soil associations classified as rendzinas. All the rendzinas are derived from chalk and limestone and have high critical loads. Of the rankers, three of the eight are also derived from chalk and limestone with correspondingly high critical loads; the remaining five rankers are derived from sandstones, mudstones, siltstones, acid or igneous rocks, and hence have much lower critical loads (0.2–0.5 keq ha^{-1} year^{-1}). Although this suggests a bi-modal distribution for the lithomorphic soils, the observation that 20 out of 25 of these soil associations are in a single class leads to it being considered a 'strong' piece of evidence for the highest critical load in the major soil group endorsement table. However, it is debateable how the evidence based on lithomorphic and ranker soils should be used. If the parent material is also known, then a rule could be applied that sets the critical load to the higher class where the parent material is limestone or chalk, or to the lower class where the soils are derived from other basic or acid parent materials. An elegant solution that deals effectively and appropriately with this situation is still needed (see Section 5).

Table 2.9. Endorsement summary for H21 *Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium* heath[1]

ET consulted	'unified' terminology	Critical load class[2]					
		CL-1	CL-2	CL-3	CL-4	CL-5	Peat
Major soil group	lithormorphic	vw	vw			s	
Soil sub-group	ranker	vw	w			w	
Soil type	humic ranker	vw	vw			vw	
Totals	very weak	3	2			1	
	weak	0	1			1	
	moderate	0	0			0	
	strong	0	0			1	
Final endorsement		very weak	very weak			likely	

[1] vw = very weak; w = weak; m = moderate; s = strong.

[2] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads.

When testing the method with the 22 NVC heaths, none of the communities were awarded a 'definitive' endorsement (Table 2.10).

In summary, the results showed:

- five heaths with the 'confident' attribution for the critical load to be in the lowest or next lowest class (i.e. critical loads ≤0.5 keq ha^{-1} year^{-1});
- six heaths with a 'likely' attribution for the critical load to be in the lowest or next lowest class (i.e. critical loads ≤0.5 keq ha^{-1} year^{-1});
- 13 heaths with a 'likely' attribution for the highest critical load (i.e. >2.0 keq ha^{-1} year^{-1}). Of these, 12 were associated with lithomorphic soils (see above) and one (H4) with a reference to growing on limestone but only in a situation where the limestone was covered in loess thereby insulating the community from the geology;

- three of the wetter more high altitude heaths are recorded as growing on peat soil and are therefore probably grading into the NVC ‘mire’ communities.

Heaths marked Y in the ‘litho’ (lithomorphic) column of Table 2.10 are those NVC communities associated with rankers or lithomorphic soils. If the weight of evidence provided by the lithomorphic attribute is removed, then none of those heaths has a ‘likely’ endorsement for a high critical load.

Only one class (H8 *Calluna vulgaris – Ulex gallii*) has the attributes of lithomorphic soils and limestone geology. It may be possible in this case that the ‘likely’ score for the highest critical loads is valid, but the community is also found on more acid bedrocks and soils.

Two of the heaths (H7 and H21) are noted as occurring on lithomorphic ranker soils but there is no information on their parent materials, resulting in ‘likely’ endorsements for the higher critical loads.

The heath H5 is associated with the Lizard Peninsular in Cornwall where the underlying geology is unique in Western Europe, being base rich but calcium poor.

Table 2.10 Endorsement[1] of critical load class for all NVC heath communities

Code	Community name	Critical load class[2]						
		CL-1	CL-2	CL-3	CL-4	CL-5	Peat	Litho
H1	*Calluna vulgaris – Festuca ovina*	weak	conf	weak	vw	weak		
H2	*Calluna vulgaris – Ulex minor*	conf	weak	vw	weak	vw		
H3	*Ulex minor – Agrostis curtisii*	conf	weak					
H4	*Ulex gallii – Agrostis curtisii*	vw	weak	weak	weak	lik		*
H5	*Erica vagans – Schienus nigricans*		weak	vw	lik	weak	lik	
H6	*Erica vagans – Ulex europaeus*	vw	weak	weak	lik	lik		Y
H7	*Calluna vulgaris – Scilla verna*	vw	vw			lik		Y
H8	*Calluna vulgris – Ulex gallii*	lik	weak	vw	vw	lik		Y
H9	*Calluna vulgaris – Deschampsia flexuosa*	lik	lik	weak	weak	lik		Y
H10	*Calluna vulgaris – Erica cinerea*	lik	weak	weak	vw	lik		Y
H11	*Calluna vulgaris – Carex arenaria*	weak	vw			vw		
H12	*Calluna vulgaris – Vaccinium myrtillus*	conf	weak	weak	vw	vw		
H13	*Calluna vulgaris – Cladonia arbuscula*	lik	weak			lik		Y
H14	*Calluna vulgaris – Racomitrium lanuginosum*	lik	weak		vw	lik		Y
H15	*Calluna vulgaris – Juniperus communis*	weak	weak		vw	lik		Y
H16	*Calluna vulgaris – Arctostaphylos uva-ursi*	conf	weak	weak	vw	vw		
H17	*Calluna vulgaris – Arctostaphylos alpinus*	weak	weak		vw	lik		Y
H18	*Vaccinium myrtillus – Deschampsia flexuosa*	weak	weak		vw	lik		Y
H19	*Vaccinium myrtillus – Cladonia arbuscula*	weak	weak		vw			
H20	*Vaccinium myrtillus – Racomitrium lanuginosum*	weak	weak		vw	lik	lik	Y
H21	*Calluna vulgaris – Vaccinium myrtillus – Sphagnum capillifolium*	vw	vw			lik		Y
H22	*Vaccinium myrtillus – Rubus chamaemorus*						lik	

[1] conf = confident; lik = likely, weak; vw = very weak; Y = yes.

[2] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads

2.6 Comparison with other datasets: Countryside Survey 2000

The endorsement methodology described above was developed based on Rodwell's description of diagnostic examples (Rodwell, 1991). In reality, vegetation communities and species are often rather more flexible and grade imperceptibly between each other.

As part of the Countryside Survey 2000 (DETR/CEH, 2000), botanic and soils data were collected from plots within 512 one-kilometre squares located across the UK according to a stratified random sample. A computer program, MAVIS (Modular Analysis of Vegetation Information System), allowed the translation between the Countryside Vegetation System (CVS) botanic records and the NVC. The soils information was also translated into the Soil Survey descriptions.

This dataset provides an alternative, independent (statistical) view of the relationship between vegetation and soils, and can provide a comparison with the results above. Due to the two translation processes, this is not a strict validation – just a comparison.

Despite the widespread nature of the Countryside Survey 2000 (CS2000), only eight of the 22 NVC heath communities were apparently encountered and, of those, only six had more than one useable example. The endorsement of the critical loads for the six heaths found more than once in CS2000 is shown in Table 2.11.

Table 2.11 Endorsement of critical load classes using data from the Countryside Survey 2000

		Critical Load Class[1]					
Code	**Number quadrats**	**CL-1**	**CL-2**	**CL-3**	**CL-4**	**CL-5**	**Peat**
H1	14	likely	weak			very weak	very weak+
H2	4	likely	weak	very weak		very weak	
H8	26	likely	weak	weak	weak		very weak+
H9	5	likely	very weak	very weak			
H10	9	very weak	weak				very weak
H12	19	likely	weak			very weak	very weak+

[1] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads

The main difference between the endorsements from CS2000 data and those from Rodwell (1991) is that many more of the heaths are observed on peat soils than expected from the description of the communities.

Three communities (H1, H8 and H12) were apparently observed on peat soil associations which are not described by Loveland (they may be sites in Scotland); hence, the temporary endorsement of ‘very weak+’ for the peat critical load. All the communities which were (unexpectedly) observed on peat are dominated by *Calluna* and it is possible that there is a difference between what a botanist and a soil scientist consider to be peat.

The impression from Table 2.11 is that the endorsed critical load class based on CS2000 data tends to be lower than the estimate from Rodwell, but there are insufficient examples to draw a firm conclusion.

The differences between the NVC and the CS2000 tell us more about how soil and vegetation are perceived in the field than it does about the Endorsement Theory approach. The comparison shows that the NVC descriptions have the widest possible range of associations between the vegetation community and its environment; this means that rare and atypical cases are still described. For example, the heaths (H4) growing over limestone are ‘protected’ from the calcium by a smear of clayey drift; this means that there is at least some possible endorsement for heaths over limestone. However, the area of the UK where there is limestone near to the surface but where it is covered in non-calcareous clay is likely to be a small fraction of 1 per cent of the total land area. Hence, the chance of the CS2000 having a sample square on that combination is very low.

The stratified random sample approach of the CS2000 will tend to record the dominant or most frequent combinations of communities and environment. In general, the range of possibilities described in the CS2000 will be narrower than the range of possibilities described in the NVC. Theoretically, the range of combinations in the CS2000 is less than or equal to the range of possibilities in the NVC.

Unfortunately, there are three reasons why this theory may not be applicable:

- There could be omissions from the NVC that are observed in the CS2000.
- Definitions vary – with the heaths, this is observable in the relationship with ‘peat’ where ‘vernacular’ usage does not necessarily correspond to that used by soil surveyors.
- Two conversion processes are needed with the CS2000 (one for plant species to NVC community and the second with the soils) and both could introduce error or bias.

2.7 Experience of applying the endorsement approach

2.7.1 Lack of data

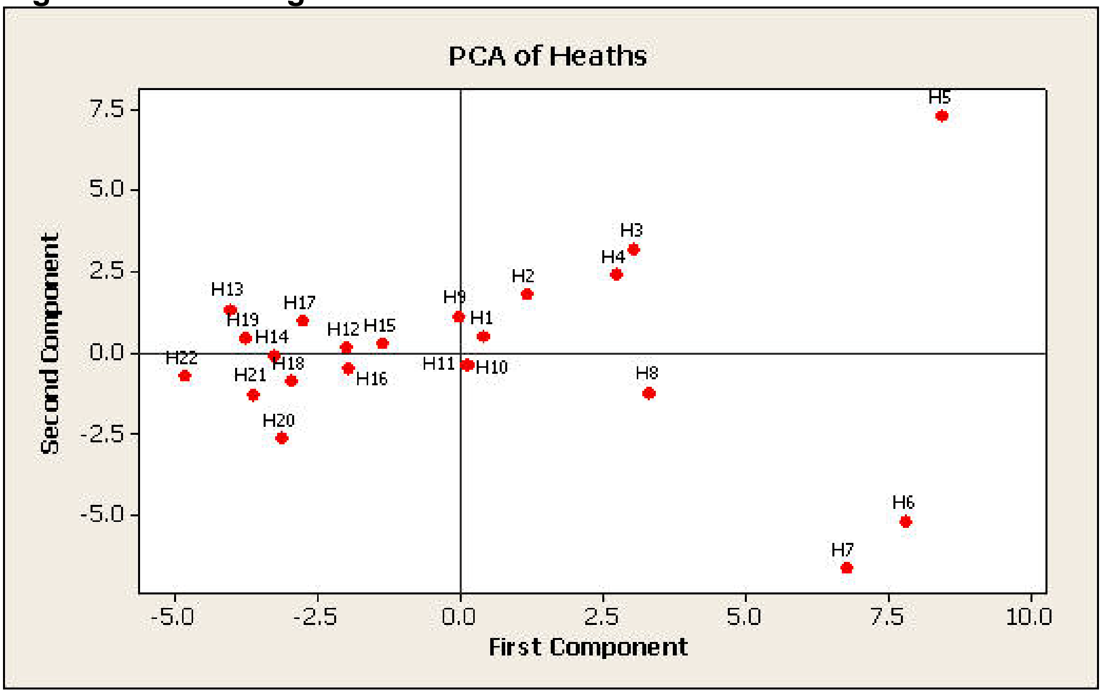
In applying the endorsement approach, it could be argued that there should be a threshold in the amount of data considered acceptable below which no endorsement should be made. In the case of the NVC heath communities, the information available for H7 and H21 (see Section 2.5) is minimal – being limited to the two or three pieces of information on soil groups/types. In such instances, the extent to which a useful endorsement can be made is debatable.

Table 2.10 shows that, for some NVC heaths, there is only a weak endorsement for any hypothesis. Rodwell (1991) uses the term 'community' to describe associations or groups of species that have similar requirements and which are able to co-exist. Like all communities, the boundaries between them can be indistinct and a site may contain a suite of species that are consistent with one, several or no described community.

> *'Vegetation being as variable as it is, it is sometimes expedient to allocate a sample to a community even though the named species are themselves absent.'* (Rodwell, 1991).

One way to examine the relationship between the communities is to use principal components analysis (PCA) with the frequency of the different plant species being the variables. Figure 2.3 illustrates this process and is based on those species that occur with a frequency of at least 60 per cent in at least one community type.

Figure 2.3 PCA diagram of the 22 NVC heath communities



The two heaths with the largest first axis score are H5 (*Erica vagens-Schoenus nigrans* heath) and H6 (*Erica vagens – Ulex europaeus* heath). However, these are confined to the Lizard Peninsular in Cornwall, an area with an unusual geology and the only place in Western Europe where Serpentine rocks are found at the surface. The next nearest, H7 (*Calluna vulgaris – Scilla verna* heath) is a strictly coastal community. At the other end of the first axis is H22 (*Vaccinium myrtillus – Rubus chamaemorus* heath), a high altitude Scottish heath.

It was hoped that either a PCA plot or a dendrogram based on the species data could be used to infer an appropriate critical load class for those communities where Rodwell records little environmental information. Unfortunately, no discernable pattern can be deduced from either representation. This is possibly because, in reality, most heaths are dominated by *Calluna* and similar species, and have a similar environmental response.

2.7.2 Precautionary principle

Applying the precautionary principle would set the critical load to the class of lowest critical load values from those for which an endorsement had been assigned. Thus, if the precautionary principle was applied rigorously to the NVC heath communities (Table 2.10), only two (H5 and H22) would not have a critical load of $\leq$0.2 keq ha^{-1} $year^{-1}$. The critical load for H22 could also be very low, as it requires the peat methodology to be applied. The extent to which a strict interpretation of the precautionary principle can be inserted into the endorsement methodology is not clear.

2.8 Using the Microsoft Access database

All the data used to apply the endorsement approach have been collated within a Microsoft Access database (ea_nvc_data.mdb). Its contents are summarised in Figure 2.1 and further information is given in Annex 4.

Two reports allow database users to access the information by NVC community (i.e. NVC class):

- a summary of the information extracted and stored for each NVC community, e.g. as shown in Annexes 1 and 2.
- a summary of the endorsement information available for each NVC community, e.g. Annex 3.

These reports are generated as follows:

(i) Open the Access database (nvc_data.mdb) provided on the CD.
(ii) Click on the Reports icon.
(iii) For the NVC summary report, double click on ncv_summary_report and then enter the code for the NVC community of interest in the pop-up box, e.g. H2 (upper or lower-case).

(iv) For the endorsement summary report, double click on nvc_endorsement_summary and enter the NVC community code in the pop-up box.

The reports can be printed directly from Microsoft Access. Alternatively, by using the File menu ‘export’ command, the report can be exported to Microsoft Word in rich text format (rtf) or to HTML format. Exporting the reports to other packages may alter the appearance of the report, e.g. by removing the horizontal linework.

NB The reports may not run correctly if changes are made to the underlying tables, queries or relationships.

3. WORK PACKAGE 2: COMBINING NATIONAL AND LOCAL ESTIMATES OF CRITICAL LOADS AND APPLYING DEMPSTER–SHAFER STATISTICS

This section examines the feasibility of combining national and local estimates of critical loads through the application of Dempster–Shafer statistics.

Both national and local data contain some level of uncertainty. Rather than simply replacing the national estimate with the local estimate, the ‘best’ estimate of the site-specific critical load is based on ‘revising’ the national estimate using as much local information as is available. The optimum combination of local and national parameters must be scientifically reliable, understandable and defensible.

When looking at the NVC plant community data, we had incomplete and sometimes inconsistent information. We therefore chose Endorsement Theory to combine the evidence. However, when combining the national estimate with community-based evidence (though there are uncertainties in both data sources), we should be able to apply Dempster–Shafer statistics to determine whether a particular allocation is *provable* given additional evidence. The problem is formulated so that the alternatives are not mutually exclusive and the combined belief for all hypotheses is unbounded.

The frequentist or classical approach to statistics says that probability is a measure of how often an event occurs when a trial is repeated a very large number of times in a system that has no memory. Under a strict frequentist view, we are not allowed to know anything about the system being investigated except the results of the trials.

This classical view of probability is dominant in scientific analysis but does not accord with general usage. For example, many people are happy to suggest a probability that the Labour party will win the next General Election, but the election is a single unique event and therefore, to a strict frequentist, a probability attached to the event has no meaning. In contrast, Bayesian statistics requires some prior knowledge about the system (a prior belief) and is concerned about whether the addition of new evidence increases or decreases our belief in a particular outcome.

Dempster–Shafer is an extension of Bayesian statistics, which introduces the concept of *uncertainty* and *plausibility* into the reasoning process. Under Dempster–Shafer, belief and disbelief can sum to less than 1: the difference between them is the uncertainty; the uncertainty plus the belief is the plausibility. Evidence may make only a weak contribution towards the belief in a hypothesis, but the hypothesis will still be plausible if the uncertainty is high and, crucially, the evidence does not have to support the negation of the hypothesis.

This is analogous to the approach taken by Endorsement Theory, where a weight of belief for an alternative does not imply a disbelief in any other option, and the total weight of belief offered by a piece of evidence does not have to 'sum' to any particular total. If there is no uncertainty, then Dempster–Shafer is mathematically equivalent to a Bayesian formulation.

In combining the local (i.e. endorsement) and the national critical loads data, we made the following assumptions:

- both strands of evidence are uncertain;
- both strands of evidence have value.

Provided we know the associated uncertainty (or the disbelief), we can use the mathematical formulations from Tangestani and Moore (2002) to combine two different beliefs.

$$Bel = [(Bel_1 \times Bel_2) + (Bel_1 \times Unc_2) + (Bel_2 \times Unc_1)] / \beta \qquad \text{(Eq 1)}$$

$$Dis = [(Dis_1 \times Dis_2) + (Dis_1 \times Unc_2) + (Dis_2 \times Unc_1)] / \beta \qquad \text{(Eq 2)}$$

$$Unc = (Unc_1 \times Unc_2) / \beta \qquad \text{(Eq 3)}$$

where:

subscripts indicate the two sources of evidence
Bel = belief
Dis = disbelief
Unc = uncertainty
β = normalising factor (ensures that Bel + Dis + Unc = 1).

$$\beta = 1 - (Bel_1 \times Dis_2) - (Bel_2 \times Dis_1) \qquad \text{(Eq 4)}$$

The possible application of this approach by combining the endorsement results from Section 2 with national estimates of critical loads and their uncertainties is described below.

3.1 Converting endorsement to numeric values

From the analysis of the NVC vegetation communities, a symbolic 'weight' (endorsement) has been assigned to each critical load hypothesis. Even where the endorsement is 'definitive', the 'evidence' is still an inference that relies on the assumption that the terms used by Loveland (1991), the Soil Survey of England and Wales (Rothamsted, 1983) and Rodwell (1991) have been translated consistently between the views of soil scientists and botanists. It also relies on the vegetation having been allocated into the most appropriate NVC community.

To apply Dempster–Shafer, numerical values for belief and uncertainty have been allocated to the endorsements (Table 3.1). Like the choice of terminology, these values reflect expert opinion and different experts could choose different terms.

Table 3.1 Conversion of endorsement to numerical values for belief and uncertainty

Endorsement	Belief	Uncertainty
Definitive	0.90	0.1
Confident	0.75	0.2
Likely	0.50	0.3
Weak	0.25	0.3
Very weak	0.10	0.4

3.2 Variance in national critical loads

The UK map of acidity critical loads for soils is based on the weathering rate and mineralogy of the dominant soil type in each 1-km grid square. The soils database for England and Wales shows that the dominant soil may occupy as little as 20 per cent of the area of the grid cell; the dominant soil for ~10 per cent of the grid squares occupy ≤50 per cent of the grid area.

A formal assessment of uncertainties in national critical loads has been carried out for Defra (Hall *et al.*, 2004). This included an examination of the effects of basing the critical loads on the dominant, most or least acid-sensitive soil in each 1-km grid square. However, this variance was not quantified.

For this study, an area of 20 × 20 km centred on Purbeck in Dorset was selected for further analysis as an area where heaths are known to occur. According to the England and Wales soils database, up to seven soil associations may be found in any 1-km square (although the arithmetic mean is very close to 2).

For each 1-km square, the critical load for the dominant soil was taken as the most likely value. It is then possible to use the critical load of the sub-dominant soils (which may or may not have the same critical load as the dominant soil) to calculate the expected variance within the square (Table 3.2).

Table 3.2 Expected variance (per cent) in national acidity critical loads for soils in the Purbeck region of Dorset

		Variance (per cent) in critical load by critical load class[1]				
		CL-1	**CL-2**	**CL-3**	**CL-4**	**CL-5**
Critical load class[1] for the dominant soil	**CL-1**	81.1	0.4	16.3	1.0	1.1
	CL-2	4.8	68.4	16.1	0.0	10.7
	CL-3	12.2	1.4	82.7	1.1	2.0
	CL-4	9.3	0.0	15.7	75.0	0.0
	CL-5	3.4	8.8	14.1	0.0	73.0

[1] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads

For example, in those squares where the dominant soil belongs to critical load class CL-2:

- 68.4 per cent of the area of the square is, on average, occupied by soils with a class CL-2 critical load (i.e. range 0.2–0.5 keq ha^{-1} $year^{-1}$);
- 16.1 per cent of the area is occupied by soils with a CL-3 critical load class (i.e. range 0.5–1.0 keq ha^{-1} $year^{-1}$);
- 4.8 per cent of the area with a CL-1 critical load class (i.e. range ≤0.2 keq ha^{-1} $year^{-1}$).

Just over 50 per cent of all the soils represented in the Purbeck region have critical loads in the range 0.5–1.0 keq ha^{-1} $year^{-1}$, a further 30 per cent of the soils have critical loads ≤0.2 keq ha^{-1} $year^{-1}$, and <10 per cent of the soils occur in each of the three other critical load classes. Note that this distribution is likely to differ from one region to another across the country.

3.3 Applying Dempster–Shafer statistics

For a site (e.g. 1-km square) in the Purbeck region, the acidity critical load based on the dominant soil can be extracted from the national map (Figure 3.1). Within the Dempster–Shafer calculation, the ‘belief’ for each critical load ‘hypothesis’ is obtained by converting the percentage from Table 3.2 into a proportion. The ‘uncertainty’ is calculated as 1 – belief and the disbelief for the national data is set to zero.

3.3.1 Example

Consider the case of a site located where:

- mean national critical load = 0.75 keq ha^{-1} $year^{-1}$ (i.e. mid-range class CL-3)
- NVC class = H1 heath.

The information on the following can now be combined and summarised as in Table 3.3:

- the endorsement of the five critical loads for H1 is given by the H1 row of Table 2.10;

- the belief and uncertainty for H1 as given by the conversion in Table 3.1, e.g. the endorsement 'weak' (as for CL-1 and CL-3) has a belief of 0.25 and an uncertainty of 0.3;

- the belief and uncertainty for the national critical loads as given by the expected variance as a percentage in Table 3.2, e.g. reading off from the row for critical load class CL-3, the belief for CL-1 is 12.2 per cent. This is given in Table 3.3 in numerical format as 0.122. The uncertainty for CL-1 is therefore (1 – 0.122) = 0.878.

Figure 3.1 Empirical critical loads of acidity for soils in the UK

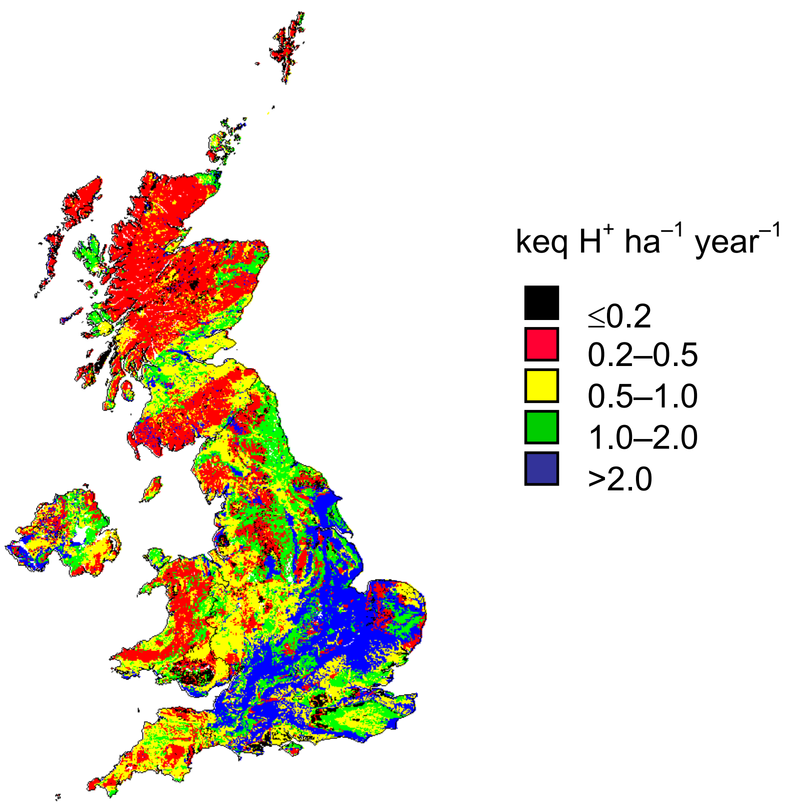


Table 3.3 Combined information for NVC heath H1 at a site with a national soil critical load of 0.75 keq ha^{-1} year^{-1} (i.e. mid-range value of class CL-3)

Critical load class[1]	From Table 2.10	From Table 3.1[2]		From Table 3.2[3]	
	Endorsement	**Bel_1**	**Unc_1**	**Bel_2**	**Unc_2**
CL-1	Weak	0.25	0.3	0.122	0.878
CL-2	Confident	0.75	0.2	0.014	0.986
CL-3	Weak	0.25	0.3	0.827	0.173
CL-4	Very weak	0.1	0.4	0.011	0.989
CL-5	Weak	0.25	0.3	0.020	0.980

[1] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads

[2] Bel_1 and Unc_1 are the belief and uncertainty from the endorsement (converting the percentage to a proportion)

[3] Bel_2 and Unc_2 are the belief and uncertainty from the national map.

$$\text{Belief (Bel)} + \text{Disbelief (Dis)} + \text{Unbelief (Unc)} = 1 \quad (5)$$

From the shaded row of Table 3.3 for CL-3:

If $Bel_1 = 0.25$ and $Unc_1 = 0.3$, then $Dis_1 = 0.45$.

$Dis_2 = 0.0$ because Bel_2 (0.827) + Unc_2 (0.173) = 1.

Applying equations (1)–(4) for the hypothesis that the critical load is CL-3 gives:

$$\begin{aligned}\beta &= 1 - (Bel_1 \times Dis_2) - (Bel_2 \times Dis_1) \quad (4)\\ &= 1 - (0.25 \times 0.0) - (0.827 \times 0.45)\\ &= 0.628\end{aligned}$$

$$\begin{aligned}Bel &= [(Bel_1 \times Bel_2) + (Bel_1 \times Unc_2) + (Bel_2 \times Unc_1)] / \beta \quad (1)\\ &= [(0.25 \times 0.827) + (0.25 \times 0.173) + (0.827 \times 0.3)] / 0.628\\ &= 0.793\end{aligned}$$

$$\begin{aligned}Unc &= (Unc_1 \times Unc_2) / \beta \quad (3)\\ &= (0.3 \times 0.173) / 0.628\\ &= 0.083\end{aligned}$$

$$\begin{aligned}Dis &= [(Dis_1 \times Dis_2) + (Dis_1 \times Unc_2) + (Dis_2 \times Unc_1)] / \beta \quad (2)\\ &= [(0.45 \times 0.0) + (0.45 \times 0.173) + (0.0 \times 0.3)] / 0.628\\ &= 0.124\end{aligned}$$

Applying the equations to the remaining critical loads hypotheses (Table 3.4) tells us the belief, uncertainty and disbelief we have in our critical load for the site actually being in class CL-3 or in any of the other classes. The highest belief (0.793) is for CL-3, so from this analysis we would not revise our estimate from the national map.

However, the belief for CL-2 (0.753) is only slightly lower and this is also the class with a 'confident' endorsement score (Table 3.3). In this case, we might suggest choosing a value at the lower end of the CL-3 range. As the critical load classes represent ranges of values, this means that the critical load for the NVC community at this example site lies between 0.2 and 1.0 keq ha^{-1} $year^{-1}$.

Table 3.4 Combined belief, uncertainty and disbelief values calculated for each critical load hypothesis for the example site

Critical load hypothesis[1]	Belief[2]	Uncertainty	Disbelief	β
CL-1	0.303	0.279	0.418	0.945
CL-2	**0.753**	0.197	0.049	0.999
CL-3	**0.793**	0.083	0.124	0.628
CL-4	0.105	0.398	0.497	0.995
CL-5	0.109	0.396	0.495	0.990

[1] See Table 2.3: Class CL-1 has the lowest critical loads and CL-5 the highest critical loads.
[2] The beliefs for all the possible hypotheses **do not sum to 1.0**.

4. CONCLUSIONS

Uncertain, incomplete and contradictory information are common occurrences in all areas of environmental science. Decision-makers and land managers increasingly want estimates specific to a particular piece of ground, but it is unrealistic to expect site-specific measured data to exist for more than a small number of sites. It is also unrealistic to expect data collected to assess national or super-national concerns to be ideal for site-specific concerns. This report documents the application of Endorsement Theory and Dempster–Shafer methods to this issue. An initial assessment suggests that this would provide the Environment Agency with a defensible, usable and useful tool.

Some more specific conclusions are given below.

- Site-specific measured data needed for the calculation of critical loads are likely to be available only for a very limited number of sites. In addition, there tends to be a lack of information on the soil–vegetation combinations present at a site or on the location of designated features.

- The National Vegetation Classification tends to be used to describe the vegetation communities within designated sites. In presenting our methodology, we have assumed that designated features can be related to NVC communities.

- The environmental information for NVC communities is variable and often incomplete. Endorsement Theory allows incomplete information to be assessed and combined in a way that makes expert opinion explicit and traceable. The reasons for the endorsement of any hypothesis can always be identified and tested against other opinions.

- Other experts may wish to set a threshold for the minimum number of pieces of evidence on which an endorsement is made.

- Endorsement Theory provides a defensible estimate of the most likely range of acidity critical loads for an NVC community. However, the range may span more than one critical load class.

- There are differences in the soils and geology terminology used by Rodwell (1991) in NVC descriptions and the terminology used by Loveland (1991) and the Soil Survey of England and Wales (Rothamsted, 1983). We have only inferred relationships where it is easy to do so; however, this remains an area of uncertainty in the methods (see Section 5).

- A lack of data on soil parent material for NVC communities can make it difficult to determine the appropriate critical load range for those communities occurring on lithomorphic soils.

- This study has demonstrated the feasibility of an endorsement approach. However, further work is required (see Section 5) before we would recommend its application in the wider community.

- The Microsoft Access database provided with this report includes information on heathland, grassland, upland/montane, mire, swamp and woodland communites. However, the results are focused only on NVC heathland communities. Other conclusions may result from the detailed study of other community types.

- The database provides the Environment Agency and other users with the means to generate reports by NVC community to summarise the environmental information held and the relevant endorsement scores.

- Section 3 on applying Dempster–Shafer statistics demonstrates how national and local information could be combined and compared in a statistically robust fashion.

- Dempster–Shafer statistics require information on uncertainty which may not always be readily available or easy to estimate.

5. FURTHER WORK

The application of Endorsement Theory to the setting of acidity critical loads for National Vegetation Classification communities has identified the following issues that require further consideration:

- Any internally consistent expert should provide similar results to those presented here. This assertion should be rigorously tested.

- We have derived the relationships between NVC soil and geology terminology and Loveland/Soil Survey terminology. Further validation and quality assurance by an expert soil scientist is recommended. Updates or revisions to the underlying tables may be necessary.

- Qualitative and symbolic approaches to reasoning are not widely employed. Although they appear to be ideally suited to this problem, it is possible that their practical application may be hindered by organisational and cultural constraints. Interaction with those responsible for site-specific critical loads would be welcomed. Comber *et al.* (2003) applied similar approaches to land cover change.

- We recommend that the methodology is tested for a selection of Special Areas of Conservation (SACs) in collaboration with experts from the Environment Agency and the conservation agencies. In particular, to consider:
  - whether designated species can be related to the NVC (many habitats designated by Annex 1 of the Habitats Directive are also described in terms of NVC communities);
  - the ease of use/application of the methodology;
  - how to deal with multiple NVC classes at a single site;
  - inclusion of the Dempster–Shafer technique.

- The extent to which Endorsement Theory is compatible or could be made compatible with a strict interpretation of the precautionary principle is uncertain.

- Testing the methodology with the NVC heath communities revealed an inconsistency within one attribute of one aspect of the environment (the lithomorphic soils). The extent to which there are other similar problems with other communities or less obvious data problems within the heath or other communities should be investigated.

- The description of NVC communities by Rodwell (1991) is based on exemplars. In reality, vegetation communities are often difficult to precisely and unambiguously allocate to these classes. The effects this could have on the endorsement of a particular hypothesis is uncertain.

- The description of the NVC communities by Rodwell (1991) is variable and not all environmental data are recorded for all communities. Although Endorsement Theory is suited to working with incomplete data, the effect of missing information cannot be quantified. Other sources of information to ‘fill the gaps’ in Rodwell’s descriptions of the environmental conditions associated with some communities should be investigated.

- Given the size and scope of the Countryside Survey 2000 (DETR/CEH, 2000), it is surprising that so few NVC heath communities were encountered. The extent to which this reflects the abundance of different communities, the very different sampling strategies employed or the translation process is unclear and should be investigated.

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Annex 1 Example report from ‘nvc_summary_report’ for H1

Note the format of this report differs slightly to that viewed in Microsoft Access due to exporting the report to Microsoft Word.

NVC class summary

H1 *Calluna vulgaris - Festuca ovina* heath

Distribution eastern England especially Breckland

Climate continental influence

Annual rainfall (mm) <650

Annual wet days <120

Rainfall pattern summer maximum (slight)

Maximum summer temp (degrees C) >30

Minimum February temp (degrees C) <0

Number frosts per year

Altitude (m): minimum 1 ***maximum*** 76 ***mean*** 30

Altitude comment

Slope (degrees): minimum ***maximum*** ***mean***

Slope comment

Topography lowland

Anthropogenic factors burning & grazing

Soil association	***Soil type***	***Comment***
	non-calcareous brown soils	
	brown sands	
Worlington		
Newport		

Soil texture sandy

Soil texture sandy-skeletal material upper horizon

Base status poor ***Nutrient status*** impoverished

Nutrient status oligotrophic

H1 *Calluna vulgaris - Festuca ovina* heath

pH: minimum ***maximum*** ***mean***

comment: soil pH acid, low surface pH

comment: water pH

Soil processes podzolisation (signs of)

Parent material naturally acidic

Parent material sometimes aeolian sand

Parent material mostly sandy glacial-fluvial drift

Parent material sometimes arenaceous

Hydrology free to excessive drainage

NVC species list	***Frequency****
Calluna vulgaris	5
Dicranum scoparium	4
Festuca ovina agg.	4
Hypnum cupressiforme sens.lat.	4
Carex arenaria	2
Cladonia impexa	2
Cladonia uncialis	2
Hypogymnia physodes	2
Rumex acetosella subsp.acetosella	2
Agrostis capillaris	1
Campanula rotundifolia	1
Cerastium fontanum subsp.vulgare	1
Cladonia arbuscula	1
Cladonia coniocraea	1
Cladonia fimbriata	1
Cladonia furcata	1
Cladonia macilenta	1

H1 Calluna vulgaris - Festuca ovina heath

Cladonia pyxidata	1
Cladonia squamosa	1
Cladonia tenuis	1
Cornicularia aculeata	1
Deschampsia flexuosa	1
Dicranella heteromalla	1
Erica cinerea	1
Erica tetralix	1
Galium saxatile	1
Holcus lanatus	1
Hylocomium splendens	1
Pleurozium schreberi	1
Polytrichum juniperinum	1
Pteridium aquilinum	1
Ptilidium ciliare	1
Scleropodium purum	1
Senecio jacobaea	1
Teucrium scorodonia	1
Ulex europaeus	1

*Frequency:
1 = scarce; 2 = occasional; 3 = common/frequent; 4 and 5 = constant in community.

Annex 2 Example report from ‘nvc_summary_report’ for H21

Note the format of this report differs slightly to that viewed in Microsoft Access due to exporting the report to Microsoft Word.

NVC class summary

H21 Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium heath

Distribution north-west Scotland, Skye, occasionally Orkney, south-west Scotland, Lake District

Climate cool but equitable

Annual rainfall (mm) 1600–3200

Annual wet days >220

Rainfall pattern

Maximum summer temp (degrees C) <22

Minimum February temp (degrees C) >0

Number frosts per year

Altitude (m): minimum 15 ***maximum*** 640 ***mean*** 289

Altitude comment

Slope (degrees): minimum 3 ***maximum*** 90 ***mean*** 34

Slope comment steep

Topography steep sunless slopes

Anthropogenic factors very sensitive to burning

Anthropogenic factors lightly grazed (some)

Soil association	*Soil type*	*Comment*
	fragmentary humic rankers	

Soil texture fragmentary

Base status ***Nutrient status***

pH: minimum ***maximum*** ***mean***

comment: soil pH

comment: water pH

H21 *Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium* heath

Soil processes

Parent material

Hydrology free draining but permanently moist

NVC species list	***Frequency****
Calluna vulgaris	5
Deschampsia flexuosa	5
Hylocomium splendens	5
Hypnum cupressiforme sens.lat.	5
Pleurozium schreberi	5
Rhytidiadelphus loreus	5
Vaccinium myrtillus	5
Blechnum spicant	4
Dicranum scoparium	4
Plagiothecium undulatum	4
Potentilla erecta	4
Sphagnum capillifolium	4
Bazzania tricrenata	3
Cladonia impexa	3
Dicranum majus	3
Erica cinerea	3
Listera cordata	3
Mylia taylorii	3
Racomitrium lanuginosum	3
Scapania gracilis	3
Solidago virgaurea	3
Anastrepta orcadensis	2
Anastrophyllum donnianum	2

H21 *Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium* heath

Breutelia chrysocoma	2
Carex bigelowii	2
Cladonia arbuscula	2
Cladonia gracilis	2
Cladonia uncialis	2
Dicranodontium uncinatum	2
Diplophyllum albicans	2
Empetrum nigrum subsp.hermaphroditum	2
Herbertus aduncus	2
Mastigophora woodsii	2
Plagiochila carringtonii	2
Pleurozia purpurea	2
Pteridium aquilinum	2
Ptilium crista-castrensis	2
Scapania ornithopodioides	2
Sphagnum quinquefarium	2
Thuidium tamariscinum	2
Trichophorum cespitosum	2
Vaccinium vitis-idaea	2
Agrostis canina sens.lat.	1
Agrostis capillaris	1
Arctostaphylos uva-ursi	1
Barbilophozia floerkei	1
Bazzania pearsonii	1
Calypogeia muelleriana	1
Campylopus flexuosus	1
Carex binervis	1

H21 *Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium* heath

Carex pilulifera	1
Cladonia bellidiflora	1
Cladonia pyxidata	1
Dactylorhiza praetermissa	1
Dryopteris filix-mas agg.	1
Festuca vivipara	1
Frullania tamarisci	1
Galium saxatile	1
Hylocomium umbratum	1
Hymenophyllum wilsonii	1
Hypericum pulchrum	1
Juniperus communis subsp.nana	1
Kurzia pauciflora	1
Kurzia trichoclados	1
Lophozia ventricosa	1
Luzula sylvatica	1
Melampyrum pratense	1
Molinia caerulea	1
Nardus stricta	1
Nowellia curvifolia	1
Oxalis acetosella	1
Plagiochila spinulosa sens.lat.	1
Ptilidium ciliare	1
Scapania nimbosa	1
Sorbus aucuparia	1
Sphaerophorus globosus	1
Sphagnum girgensohnii	1

H21 *Calluna vulgaris - Vaccinium myrtillus - Sphagnum capillifolium* heath

Sphagnum russowii	1
Sphagnum tenellum	1
Succisa pratensis	1
Tritomaria quinquedentata	1
Vaccinium uliginosum	1
Viola riviniana	1

*Frequency:
1 = scarce; 2 = occasional; 3 = common/frequent; 4 and 5 = constant in community.

Annex 3 Endorsement summary for H1 generated from ‘nvc_endorsement_summary’

Note the format of this report differs slightly to that viewed in Microsoft Access due to exporting the report to Microsoft Word.

NVC endorsement summary

H1 Calluna vulgaris - Festuca ovina heath

	Critical load class					
Attribute	*CL-1*	*CL-2*	*CL-3*	*CL-4*	*CL-5*	*Peat*
Major soil group						
brown soil	**VW**	**W**	**W**	**VW**	**W**	
brown soil	**VW**	**W**	**W**	**VW**	**W**	
Soil sub-group						
brown earth		**W**	**M**	**VW**	**VW**	
brown sand		**S**				
Soil association						
Newport 4		**S**				
Worlington		**S**				
Texture						
sand	**M**	**W**			**VW**	
sand	**M**	**W**			**VW**	
Parent material						
sand/sandstone	**M**	**W**		**VW**		
drift (with sandst or peat)	**W**	**W**	**VW**		**VW**	
sand/sandstone	**M**	**W**		**VW**		

Key

Critical load classes (ranges in keq/ha/year):
CL-1 <=0.2, CL-2 0.2–0.5, CL-3 0.5–1.0, CL-4 1.0–2.0, CL-5 > 2.0
Endorsements: VW = very weak, W = weak, M = moderate, S = strong

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Annex 4 Description of Microsoft Access database (ea_nvc_data.mdb)

The tables in the database are listed below. Reference should also be made to Figure 2.1 of the main report for information on the flow of data and relationships between the different datasets.

Please note that changes should not be made to the structure or content of the data tables (or 'queries' or 'reports') without first consulting the Centre for Ecology and Hydrology (CEH), as changes may prevent the queries or reports working correctly.

1. Soils data

These tables document the soils information extracted from Loveland (1991) and used by the Department of Environment's Critical Loads Advisory Group in developing the acidity soils critical loads map for England and Wales (and subsequently Scotland and Northern Ireland). The soil information includes the major group, sub-groups etc.

Table	**Description**
cl_class_list	LUT critical load classes and ranges
nsri_ew_distinct_soil	soil information

2. NVC data

These tables contain the information extracted from the Rodwell volumes for each NVC community considered in this study (see Tables 2.1 and 2.2 of main report).

NVC data table	**Description**
nvc_altitude	minimum, maximum, mean altitude and comments
nvc_anthropogenic	anthropogenic factors
nvc_base_status	base status
nvc_climate	climate, rainfall, temperature, etc.
nvc_distribution	distribution
nvc_geol_process	geological processes
nvc_hydrology	hydrology

NVC data table	Description
nvc_list	NVC community list and names (key table)
nvc_nutrient_status	nutrient status
nvc_parent_material	parent materials
nvc_slope	minimum, maximum, mean slope and comments
nvc_soil_ph	minimum, maximum, mean pH and comments
nvc_soil_processes	soil processes
nvc_soil_type	soil descriptions (associations, groups, etc.)
nvc_species_list	NVC community species list and frequency
nvc_texture	soil texture
nvc_topography	topography

The following tables are extensions to the above and include additional information after using the NVC-Soils LUTs (see below).

Table	Description
nvc_nsri_all_soils	Soil Survey/Loveland soils data added to NVC
nvc_nsri_all_texture	as above for soil texture
nvc_nsri_all_parent_material	as above for soil parent materials

3. Endorsement tables

These tables contain the endorsement scores used for the different environmental parameters.

Table	Endorsements for:
endorse_soil_association	soil associations for England and Wales
endorse_soil_subgroup	soil sub-groups
endorse_soil_major_group	soil major groups
endorse_soil_type	soil types
endorse_soil_comment	comments on soil types (not used)
endorse_texture	soil texture
endorse_parentmaterial	soil parent materials

4. NVC-Soil Look-up-tables (LUT)

These tables used to link the distinct pieces of soils information for the NVC communities to the Soil Survey/Loveland soil descriptions used for applying the endorsement theory.

Table	Description
nvc_nsri_distinct_soil_types	distinct soil descriptions
nvc_nsri_distinct_soil_texture	distinct soil texture descriptions
nvc_nsri_distinct_parent_material	distinct soil parent material descriptions

5. Queries

The queries provided in the database should not be changed or deleted. Many of these are used in the generation of the reports.

6. Reports

There are a number of reports, each based on a particular database query. The main reports consist of a number of these reports embedded together. Hence, no changes should be made to the reports provided.

The two main reports are:

- nvc_summary_report
- nvc_endorsement_summary

Instructions for their use are given in Section 2.8 of the main report.

Annex 5 Conversion of frequency to an endorsement

Tables 2.5 and 2.6 in the main report show the strength of the endorsement or weight of evidence in terms of ‘very weak’, ‘weak’, ‘moderate’ and ‘strong’ endorsements for various given relationships between the total number of events and the number of events that occurred in a single critical load class. For example, the expert allocated a ‘moderate’ endorsement if all five instances of a phenomenon occurred in a single critical load class, but a ‘strong’ endorsement if all six instances of a phenomenon occurred in a single class.

The relationships in Tables 2.5 and 2.6 were based on a qualitative opinion rather than being explicitly calculated. It is, however, possible to make some assumptions and then calculate a probability for any combination of number of classes, total number of events and number of events in a class.

In the case of critical loads, the width of the classes varies (and in one case is open ended) and their relative frequency (geographic area they cover) varies. For example, an expert might argue that five out of five is a ‘moderate’ endorsement if it occurred in the third critical load class (wide thematic range 0.5–1.0, and geographically very common), but a ‘strong’ endorsement if it occurs in the lowest critical load class (narrower thematic range 0.0–0.2 and geographically more restricted). An expert who felt that the strength of the endorsement varied with the characteristics of the classes would be required to generate six pairs of tables corresponding to Tables 2.5 and 2.6 – each pair corresponding to a single critical load class. It is not clear whether this added complexity would improve the analysis and, for convenience, each class is considered equally likely in the calculation below.

Notation:

B = number of bins or classes
M = total number of events
N = number of events in a particular bin

Assuming each bin is equally likely to be chosen, the probability that an event falls in bin ‘a’ is:

$$P(a) = 1/B \qquad \text{(Eq 1)}$$

And the probability that the event falls in some bin other bin ‘¬a’ (‘not a’) is:

$$P(\neg a) = [(1 - P(a)] = (B-1)/B. \qquad \text{(Eq 2)}$$

Treating the allocation of the M events as a sequential trial gives us a sequence of allocations to ‘a’ and ‘¬a’. If there were, say, seven events (M = 7), we might get a sequence like:

$$a, \neg a, \neg a, \neg a, a, \neg a, \neg a. \qquad \text{(Eq 3)}$$

As events are random, the probability of getting that particular sequence is:

$$P(a).P(\neg a).P(\neg a).P(\neg a).P(a).P(\neg a).P(\neg a) \qquad \text{(Eq 4)}$$

Simplifying equation 4 by gathering similar terms gives in this particular case:

$$P(s) = P(a)^2.P(\neg a)^5 \qquad \text{(Eq 5)}$$

That is for any ‘N’ and ‘M’:

$$P(s) = P(a)^N.P(\neg a)^{(M-N)} \qquad \text{(Eq 6)}$$

However, there are many ways that a trial like that in equation 3 would lead to the same total allocation to ‘a’. For example, the trial: ¬a,¬a,¬a,¬a,a,a,¬a gives the same end result. Therefore, we need to be able to calculate the number of combinations of N events out M,. This is given by:

$$C = \frac{M!}{N!(M-N)!} \qquad \text{(Eq 7)}$$

where ! is factorial, that is:

$$M! = M \times (M-1) \times (M-2) \times \ldots \times 1.$$

Thus, the probability of having exactly N events falling in a particular bin is the product of equations 6 and 7 is given by:

$$P() = \frac{M\,!}{N\,!\,(M\ -\ N\,)!}\ (1/B)^N\ ((B-1)/B)^{(m-n)} \qquad \text{(Eq 8)}$$

Worked example
What is the probability of getting five events out of a total of seven in a single class when there are six classes?

Setting:

N = 5
M = 7
B = 6

Applying equation 7 (and simplifying by cancelling common terms) gives:

$$C = \frac{M!}{N!(M-N)!}$$

$$= \frac{(7.6.5.4.3.2.1)}{(5.4.3.2.1) \times (2.1)}$$
$$= \frac{7.6}{2} \qquad \text{(Eq 9)}$$

Applying equation 6 and substituting in equations 1 and 2 gives:

$$P(s) = P(a)^{N}.P(\neg a)^{(M-N)}$$
$$= (1/6)^{5}.(5/6)^{2}$$
$$= 5^{2}/6^{7} \qquad \text{(Eq 10)}$$

Combining equations 9 and 10, and simplifying (cancelling out a 6 top and bottom) gives:

$$P() = \frac{7.5.5}{2.6.6.6.6.6.6}$$
$$= 0.00187 \qquad \text{(Eq 11)}$$

With small probabilities such as 0.00187 (0.187 per cent), it is sometimes easier to compare them if they are inverted to form probabilities in the form '1 in x'. In this case, 0.00187 = 1:533, i.e. there is less than a one in five hundred chance of getting five events out of a total of seven in a single specified bin where there are six equally probable bins.

The expert is still left with the decision as to what level of probability confers strong, moderate or weak endorsement to a particular combination. From the original table of endorsements it can be inferred that the expert is intuitively using a rule base something like the one given in the table below.

Inferred rules for relating probability to strength of endorsement

Endorsement	Lower limit	Upper limit
Very weak	–	1:10
Weak	1:10	1:200
Moderate	1:200	1:10,000
Strong	1:10,000	–

For example; the probability of all five instances falling in a particular class has a probability of 1:7776 and therefore implies a 'moderate' endorsement. The probability that all six instances falling in a particular class is 1:46,656 and therefore implies a 'strong' endorsement.

Only 13 out of 105 'strength of endorsement' assignments in the original tables are inconsistent with the inferred rules. The database has been updated to be consistent with the inferred rules. An exception has been made for very infrequent events.

As the total number of events increases, the probability of getting just a few items in a class becomes very low. As an extreme example, if there are 50 events (and six equally probable classes), the chance of **not** having an event in a class is 1:9,100, and the probability of having just one observation is 1:910. Although these are highly improbable occurrences, they are not used to express a negation in the attribution.